INDEX

0042
0212
0815
1086
1178
1190
1484
2545
2578

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EXTERNALLY PRESSURIZED, AXISYMMETRICAL FOIL BEARINGS

by
Edward Barlow

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E. J. Barlow Date 4/19/63
Prepared By

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ABSTRACT

In this report, an externally pressurized axisymmetrical foil bearing was analyzed in order to obtain the qualitative effect of side leakage in a foil bearing. One or two circumferential pressure sources, symmetrically placed, were used. In the analysis, bending stiffness of the foil was included and was found to be not negligible. Curves were obtained which show the effects of the various physical parameters on the gap profile.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

NOMENCLATURE

b	half-length of the foil cylinder (in.)
b'	distance of source from foil center line (in.)
b_1	b'/b
c	unstretched circumference of foil (in.)
h	gap width between the bearing surfaces (in.)
h_o	gap width under foil center line (in.)
p	absolute pressure in lubricating air film (psi)
p_a	ambient pressure (psi)
p_1	absolute pressure at source (psi)
r	radius of foil cylinder (in.)
t	foil thickness (in.)
w	deflection of foil from gap h_o (in.)
x	axial coordinate measured from foil center line (in.)
A	flexibility parameter, $\frac{12(1-\nu^2)b^4}{R^2t^2}$
B	edge curvature parameter, t/h_o
C	dimensionless parameter, $\frac{p_aR^2}{Et}$
C_1-C_4	constants of integration
D	bending modulus, $\frac{Et^3}{12(1-\nu^2)}$ (lb-in.)
E	modulus of elasticity (psi)
F	dimensionless tension parameter, $\frac{T_oR}{h_oEt}$

G	flow rate parameter, $\frac{12\mu b R^2}{\rho_a E t h_o{}^3} Q$
H	h/h_o
K_1-K_5	constants of integration
P	p/p_a
P_1	p_1/p_a
Q	flow rate (lb mass/sec)
R	radius of rigid cylinder (in.)
T_o	tension at foil center line for gap h_o (lb/in.)
V	shearing stress resultant (lb/in.)
W	w/h_o
X	x/b
α	$\sqrt[4]{A/4}$
β	αb_1
η	λ/α
η_1-η_5	possible real and imaginary parts of η
λ	index needed for solving differential Equation (26)
λ_k	possible values for λ
μ	viscosity of air (lb-sec/in.^{2})
ν	Poisson's ratio for foil
ρ_a	density of air at ambient pressure (lb mass/in.^{3})

I. INTRODUCTION

A bearing with at least one wall made of a thin flexible material is known as a foil bearing. Actually, in all bearings, both surfaces are flexible to some extent. Thick-walled bearings have been analyzed by assuming rigid surfaces; and thin-walled bearings have been analyzed by assuming perfectly flexible surfaces (i.e., surfaces having no bending stiffness). These two limiting cases are equivalent to the limiting cases of infinite thickness and zero thickness of the bearing wall. In this report, the thickness will be considered as small but not negligible.

USES OF FOIL BEARINGS

In the most general sense, a foil bearing is used whenever a moving foil or thin sheet (commonly called a "web" in the paper industry) has to be supported. Because of the light weight of most foils, a simple stationary support is often adequate. If, in addition to support, a change in the direction of motion of the foil is desired, the friction of a stationary support is likely to be too high. In this case, a support that moves with the foil (rotating support) or a lubricating film can be used. The stationary and rotating types of supports, without lubrication between the foil and the support, have been widely used in the paper, textile, plastics, and metal industries, and also on magnetic information storage devices such as tape recorders. To the author's knowledge, lubricating films between foil and stationary guides have only been used in tape recorders, metal processing[1], and in test equipment for loading rotating shafts[2]. In all these cases, the equipment operates in air, and atmospheric air is therefore the lubricant.

Foil bearings can also be used to support entire rotors, replacing either ball or rigid surface fluid bearings for this purpose. The advantages of foil bearings are that they use fluid films for developing forces and therefore have the low friction, low noise, and long life of rigid surface fluid film bearings; but, like ball bearings, they do not have the self-excited whirl problem common to rigid surface fluid film bearings. It is certain that as the behavior of foil bearings becomes better understood and design tools other than trial and error become available, many additional uses for these bearings will be found.

PROBLEMS OF ANALYSIS

Analysis of foil bearings involves simultaneous solution of the Reynolds equation (relating the pressure in the fluid film to the parameters of that film) with the foil equilibrium equations. The resulting equation is a sixth-order, nonlinear, nonhomogeneous differential equation, for which no solution has been found. Because of the high order of the equation, finite difference solutions are difficult, and emphasis has therefore been on approximate solutions. However, by treating the problem as a one-dimensional problem and assuming the foil to be a perfectly flexible membrane, the differential equation can be reduced to a third-order nonlinear differential equation.

It should be noted that even the rigid surface problem is difficult when the lubricating film is a gas. The state-of-the-art of foil bearings is approximately that of gas bearings seven years ago; i.e., some approximate solutions have been obtained, and some understanding of their behavior has been gained.

RESULTS OF PREVIOUS STUDIES

Because of the difficulty of analysis and the limited number of applications, few studies have been made of foil bearings, and all of these have been of self-acting bearings. Blok and Van Rossum[3], who did a perturbation study of oil-lubricated foil bearings, found that the gap and the pressure were approximately constant. However, their experiments showed that cavitation occurred in the exit flow region. In their study, as well as in perturbation analyses of foil bearings by Langlois and Koh[4] and Gross[5], the foil was assumed to be perfectly flexible and infinitely long in the axial direction. These two assumptions greatly simplified the foil equilibrium equation, and also permitted axial flow and axial variations in gap and pressure to be neglected. However, the foil cannot be considered perfectly flexible whenever the second derivative of curvature is of the same order as the ratio of pressure to bending stiffness. This condition will occur in the entrance and exit flow regions.

PURPOSE OF THE PRESENT STUDY

In a finite bearing, some of the flow is in the axial direction and is lost at the side edges. This side leakage causes the pressure and the gap to decrease from that predicted for infinitely wide bearings. In the present study, this problem was formulated in order to obtain a qualitative idea of the effect of side flow and finite foil stiffness. The results of this study can also be applied to externally pressurized foil bearings in regions with primarily axial flow and almost constant circumferential curvature.

In order to eliminate flow and variations in gap and pressure in the circumferential direction, the foil is assumed to be wrapped completely around a cylinder, with a constant pressure circumferential line source placed under the foil center line in order to provide axial flow (Figure 1, with $b' = 0$). The pressure drop to ambient at the edge is expected to cause a similar decrease in gap from the center to the edge. Near the edges, the bending stiffness of the foil will cause it to be concave upwards. This anticlastic curvature effect is always found in the bending of thin plates[7].

II. ANALYSIS AND SOLUTIONS

A. FORMATION OF THE PROBLEM FOR ONE SOURCE

A line source of strength P_1 atmospheres is placed around a cylinder of radius R. A thin foil of initial length c, width 2b, and thickness t, is wrapped around the cylinder so that the line source is under the center line of the foil (Figure 1, with b' = 0).

The gap h_o under the center line is related to the circumferential tension T_o along the center line by the formula,

$$h_o = \frac{c}{2\pi}\left(1 + \frac{T_o}{Et}\right) - R, \tag{1}$$

where E is Young's modulus for the foil. The foil is assumed to be made of an isotropic material with Poisson's ratio ν. Since the foil is thin and the deflections from the circular cylindrical shape are of the same order as the thickness, we can use the plane stress approximations to the elastic equations. Under these assumptions, the equilibrium equations for the foil are the same as those for symmetrical cylindrical shells[8]. Thus,

$$\frac{Et^3}{12(1-\nu^2)} \cdot \frac{d^4w}{dx^4} + \frac{Et}{R^2}\, w = \frac{T_o}{R} - (p-p_a) \tag{2}$$

$$\frac{dN_x}{dx} = q_x = p\,\frac{dw}{dx} \tag{2a}$$

where w is the deviation from the cylinder $r = R + h_o$, and where N_x is the tangential stress resultant and q_x is the component of distributed external loading in the x-direction. The loading q_x is small except near the edge, where the slope can be large, but N_x must decay to zero at the edges, which are unconstrained. Thus, the normal component of N_x can be neglected, as was done in Equation (2). Equation (2a) can be used to find N_x, once q_x has been determined.

Since the problem has axial symmetry, we are concerned with one-dimensional flow. The governing equation of the air film will be the Reynolds equation for two surfaces that are stationary with respect to each other. Thus,

$$\frac{d}{dx}\left(h^3 p\,\frac{dp}{dx}\right) = 0 \tag{3}$$

Equations (2) and (3), together with appropriate boundary conditions, are sufficient for finding the gap and pressure profiles.

The boundary conditions due to symmetry about the foil center line are, at x = 0:

$$\frac{dw}{dx} = 0, \qquad V = \frac{d^3w}{dx^3} = 0 \tag{4}$$

The pressure and free edge conditions give us, at x = b:

$$\frac{d^2w}{dx^2} = -\frac{\nu}{R}, \qquad \frac{d^3w}{dx^3} = 0, \tag{5}$$

and

$$\frac{Et^3}{12(1-\nu^2)} \cdot \frac{d^4w}{dx^4} + \frac{Et}{R^2} w = \frac{T_o}{R} \tag{6}$$

At x = 0,

$$\frac{Et^3}{12(1-\nu^2)} \cdot \frac{d^4w}{dx^4} + \frac{Et}{R^2} w = \frac{T_o}{R} - p_a(P_1 - 1) \tag{7}$$

The conditions of Equation (5) result from the vanishing of the moment and shear resultants at the edge, and those of Equations (6) and (7) result from the given pressures, P_1 (p_1/p_a) atmospheres at the center line, and ambient pressure p_a at the edge.

Making the substitutions

$$H = \frac{h}{h_o}, \qquad W = \frac{w}{h_o}, \qquad X = \frac{x}{b}, \text{ and } P = \frac{p}{p_a} \tag{7a}$$

in Equations (2) through (7), we obtain

$$(H^3 P \cdot P')' = 0 \tag{8}$$

$$W^{iv} + AW = A\,[F-C(P-1)] \tag{9}$$

$$W'(0) = 0, \qquad W'''(0) = 0 \tag{10}$$

$$W''(1) = -\frac{\nu}{\sqrt{3(1-\nu^2)}}\sqrt{\frac{A}{4}}\,B, \qquad W'''(1) = 0 \tag{11}$$

$$W^{iv}(1) + AW(1) = AF \tag{12}$$

$$W^{iv}(0) + AW(0) = A\,[F-C(P_1-1)] \tag{13}$$

In the above equations, the prime represents differentiation with respect to X, and A, B, F, and C are given by

$$A = \frac{12(1-\nu^2)b^4}{R^2t^2}, \quad B = \frac{t}{h_o}, \quad F = \frac{T_oR}{Eth_o}, \quad C = \frac{p_aR^2}{Eth_o} \tag{14}$$

In the present form, Equations (8) and (9) are difficult to solve unless we perturb about the incompressible solution. If P_1 is of order $1 + \delta$, where $\delta << 1$, then a one-term perturbation is satisfactory. Since in most self-acting foil bearings, T/p_aR is much less than one, the perturbation method can be used, keeping only the lowest order terms, which correspond to the incompressible solution.

If the lubricating film is incompressible, Equation (8) reduces to

$$H^3P' = -\frac{G}{C} \tag{15}$$

where $-G/C$ is the constant coming from the first integration of Equation (8) and is related to the flow rate Q by

$$\frac{G}{C} = \frac{12\mu b}{\rho_a h_o{}^3 p_a}\,Q \tag{16}$$

Equations (9) and (15) can be combined, yielding

$$W^{V} + AW' = \frac{GA}{(1-W)^3} \tag{17}$$

B. APPROXIMATE SOLUTIONS

Two approximate solutions are of interest: (1) for very thin foils, and (2) for $W^2 << 1$.

1. <u>Solution for Very Thin Foils</u>

When t is small, A is large, and Equation (17) can be reduced to:

$$W' = \frac{G}{(1-W)^3} \tag{18}$$

Integration of Equation (18) gives

$$(1-W)^4 - (1-W_e)^4 = 4G\,(1-X), \tag{19}$$

where W_e is the deflection at $X = 1$. The boundary conditions are:

$$W(0) = -\,C(P_1-1) + F \tag{20}$$

and

$$W(1) = W_e = F \tag{21}$$

By definition of W and h_o, we have

$$W(0) = 0 \tag{22}$$

Evaluating Equation (19) at X = 0 gives

$$1 - (1-W_e)^4 = 4G \tag{23}$$

This solution is expected to be valid everywhere except at the center line, X = 0, and near the edge, X = 1. In those two regions $W^{iv} = 0(AW)$, and the conditions for the derivatives of W are not satisfied. Gap profiles for several values of W_e are shown in Figure 2.

Equations (18) and (22) give, at $X = 0^+$,

$$W' = G \qquad (24)$$

Since the flow is outward from the source, the gap is symmetrical about the W-axis. Thus for $X = 0^-$,

$$W' = -G \qquad (25)$$

That is, the perfectly flexible foil has a crease along its center line. Since the perfectly flexible foil has no bending stiffness, the anticlastic curvature does not occur at the edge (compare with Figures 8 through 14).

2. Solution for $W^2 \ll 1$

When $W^2 \ll 1$, the right hand side of Equation (17) can be expanded, giving

$$W^{v} + AW' \approx GA\,(1+3W) \qquad (26)$$

The solution of Equation (18) has the form

$$W = \left(\sum_{K=1}^{5} C_k e^{\lambda_k \alpha X} \right) - \frac{1}{3} \qquad (27)$$

where the λ_k's represent the roots of

$$\lambda^5 + 4\lambda - \frac{12G}{\alpha} = 0, \qquad (28)$$

and

$$\alpha = \sqrt[4]{\frac{A}{4}} \qquad (29)$$

Since Equation (28) has one real root (see Figure 3), Equation (27) can be rewritten as follows:

$$W = -\frac{1}{3} + K_1 e^{\eta_1 \alpha X} + e^{\eta_2 \alpha X}(K_2 \sin\eta_3 \alpha X + K_3 \cos\eta_3 \alpha X)$$

$$+ e^{\eta_4 \alpha X}(K_4 \sin\eta_5 \alpha X + K_5 \cos\eta_5 \alpha X) \tag{30}$$

where the η_i's are real and have been formed from the λ_k's.

Figure 3 shows the η_i's as functions of G/α. The boundary conditions, Equations (10) through (13) and Equation (22), determine K_1 through K_5 and G, respectively. However, we have seven equations and six unknowns, allowing us to use one equation, Equation (13), as a relation between T_0 and P_1. Since the η_i's are dependent upon G, the boundary equations are transcendental. For this reason in the computer programs for this problem, G was assumed to be known, and T_0 and P_1 were calculated. (For calculations see Appendix I.) Gap profiles have been plotted only for the two-source problem, of which this problem can be considered a special case.

C. EXTENSION TO TWO SOURCES

As an extension of the above problem, two circumferential line sources, located at $x = \pm b'$, were used. There is no flow between these sources, and the equation

$$W^{iv} + AW = A[F-C(P_1-1)] \tag{31}$$

holds in the edge regions. For $x > b_1$, we have $X > b_1$, where $b_1 = b'/b$, and Equation (17) is the governing differential equation for W. In addition to the boundary conditions of Equations (10), (11) and (22), we have the following continuity conditions at $X = b_1$:

$$W(b_1^-) = W(b_1^+), \quad W'(b_1^-) = W'(b_1^+),$$

$$W''(b_1^-) = W''(b_1^+), \quad W'''(b_1^-) = W'''(b_1^+) \tag{32}$$

Equation (12) is replaced by

$$W^{iv}(b_1^+) + AW(b_1^+) = A[F-C(P_1-1)] \tag{33}$$

The approximate solution developed in Equations (26) through (30) can be used in the region $X > b_1$. The equations for the constants of integration are developed in Appendix I. The curves relating T to G, with parameters B and A, are shown in Figures 4 through 7, each figure representing a different source position. Some typical profiles are shown in Figures 8 through 14.

For more than two sources, a numerical integration procedure is necessary. A computer program for numerical integration of Equation (17) for an arbitrary number of sources has been developed. By using this program we can adjust the number of sources and their relative strengths to give any desired flow rate profile.

III. RESULTS AND CONCLUSIONS

If the foil were perfectly flexible ($A \rightarrow \infty$), the foil tension T_o would be related to P_1 by

$$p_a (P_1 - 1) = \frac{T_o}{R} - \frac{Etw}{R} \tag{34}$$

The bending stiffness of the foil provides some normal shear, giving us Equation (7). However, the left hand side of Equation (7) will be small, so that P_1 and T_o are related by Equation (34), except for higher order terms. Thus of P_1, T_o, h_o, and Q, we can specify h_o and one of the others, and can solve for the remaining variables. Given P_1 and h_o, or T_o and h_o, we can also adjust Q by using two sources and varying their distance from the foil edges. Within the limitations mentioned, P_1 and T_o can be adjusted to determine h_o. As the tape stiffness increases, the edge curling effect becomes more pronounced, as is shown in Figures 12, 13, and 14. In Figures 12 and 13, the effect of increasing thickness is shown for two source positions; in Figure 14, the modulus of elasticity is varied.

The flexibility of the foil is determined by the relative importance of the term

$$\frac{Et^3}{12(1-\nu^2)} \cdot \frac{d^4w}{dx^4} \tag{35}$$

compared to the terms

$$\frac{1}{R}\left(T_o - \frac{Etw}{R}\right) , \; p_a - p \tag{36}$$

in Equation (2). As the term (35) becomes negligible, the foil becomes a membrane. Equation (2) can be transformed into the dimensionless form given by Equation (9). The flexibility parameter in Equation (9) is represented by A. As A increases, the foil becomes more flexible. Another influence on the flexibility is the edge curvature parameter B. Decreasing B increases the foil flexibility by decreasing the curvature change. The bending term becomes important if A becomes small, if the higher derivatives of W are large compared to W, or if a combination of these two effects occurs. Examining A (Equation 14), we see that decreasing the thickness, decreasing the radius of the cylinder, or increasing the foil width will make the foil more flexible. Decreasing the thickness or increasing the central gap also decreases B (Equation 14), again making the foil more flexible. A good comparison can be made using Figures 4 through 11. First, F, B, and A must be calculated from the physical parameters.(F and $C[P_1-1]$ are sufficiently close to allow one curve to represent both parameters.) Using each set of curves corresponding to a given source location (Figures 4 through 7), we find the proper value for G. The gap profile can then be found using Figures 8 through 11, with interpolation for G. Or, it can be calculated using Equations (1a) and (30), with C_1, C_2, and K_1 through K_5 given by Equations (3a), (4a), and (6a) through (10a) in Appendix I, where the η_i's can be calculated by the method shown in Appendix II.

As the sources are moved nearer the edges, we find from Figures 8 through 11 that the effect of flow rate on the gap profile diminishes. This result is due to the increased effect of foil stiffness near the edges, since the gap tends to remain nearly constant between the sources and resists rapid changes outside that region. The central gap h_0 can be varied by varying T_0 and P_1. Increasing T_0 tends to decrease h_0, while increasing P_1 tends to increase h_0.

Since, in a self-acting foil bearing, the side flow varies from zero under the foil center line to a maximum at the edges, using more than one source allows us to improve our approximation to the side flow. The major limitation on the approximate solution presented is that for the assumption $W^2 << 1$ to be valid, the minimum-to-central gap ratio cannot be greater than 0.5. The solutions presented are useful in any externally pressurized foil bearing which has an almost constant radius and little circumferential flow.

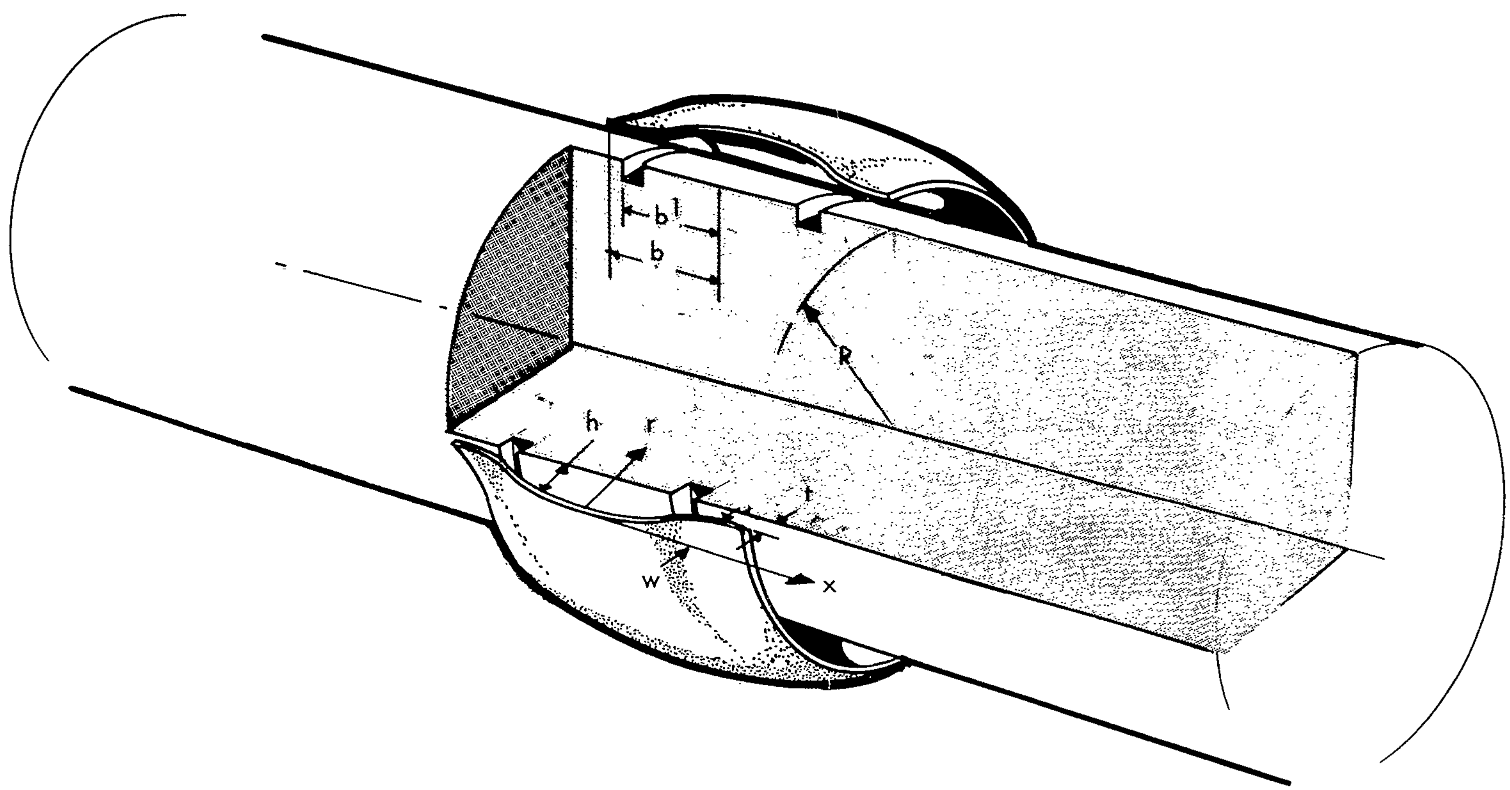


Figure 1. AXISYMMETRICAL FOIL BEARING

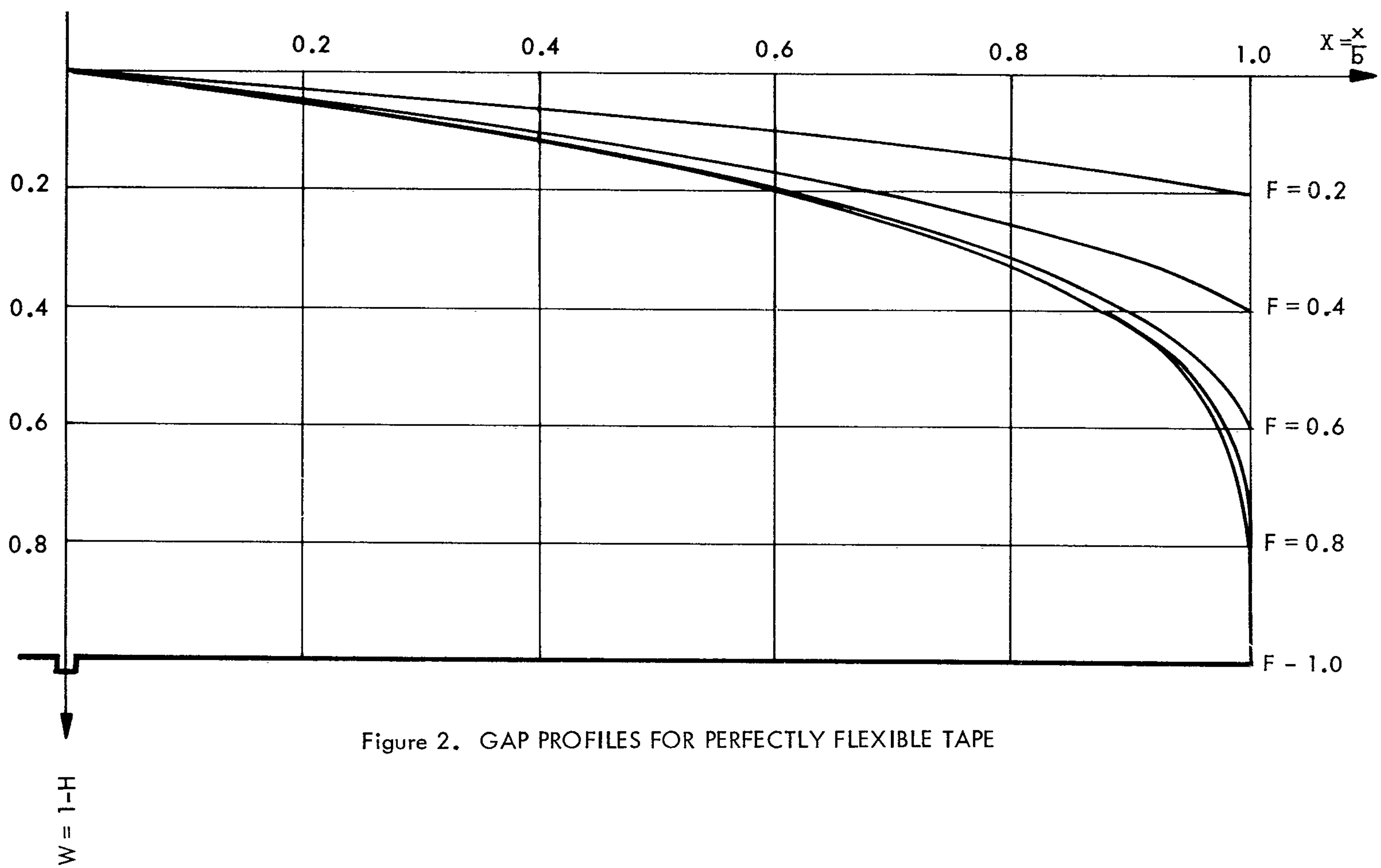
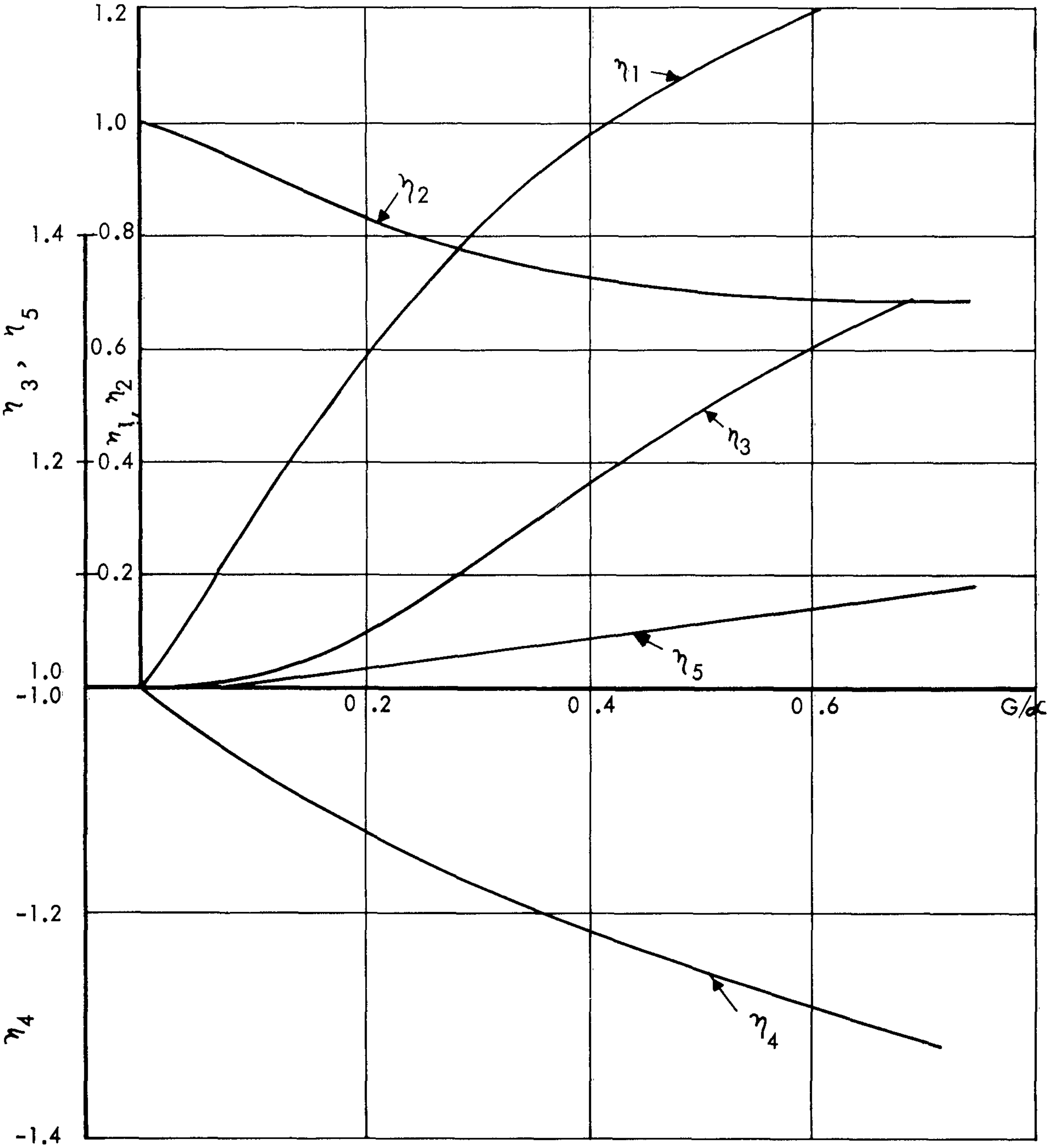


Figure 2. GAP PROFILES FOR PERFECTLY FLEXIBLE TAPE



$\eta_1; \eta_2 \pm i\eta_3, \eta_4 \pm i\eta_5$
ARE ROOTS OF $\eta^5 + 4\eta - 12\, G/\alpha = 0$.

Figure 3. VARIATION OF THE INDICIAL ROOTS AS FUNCTIONS OF G/α

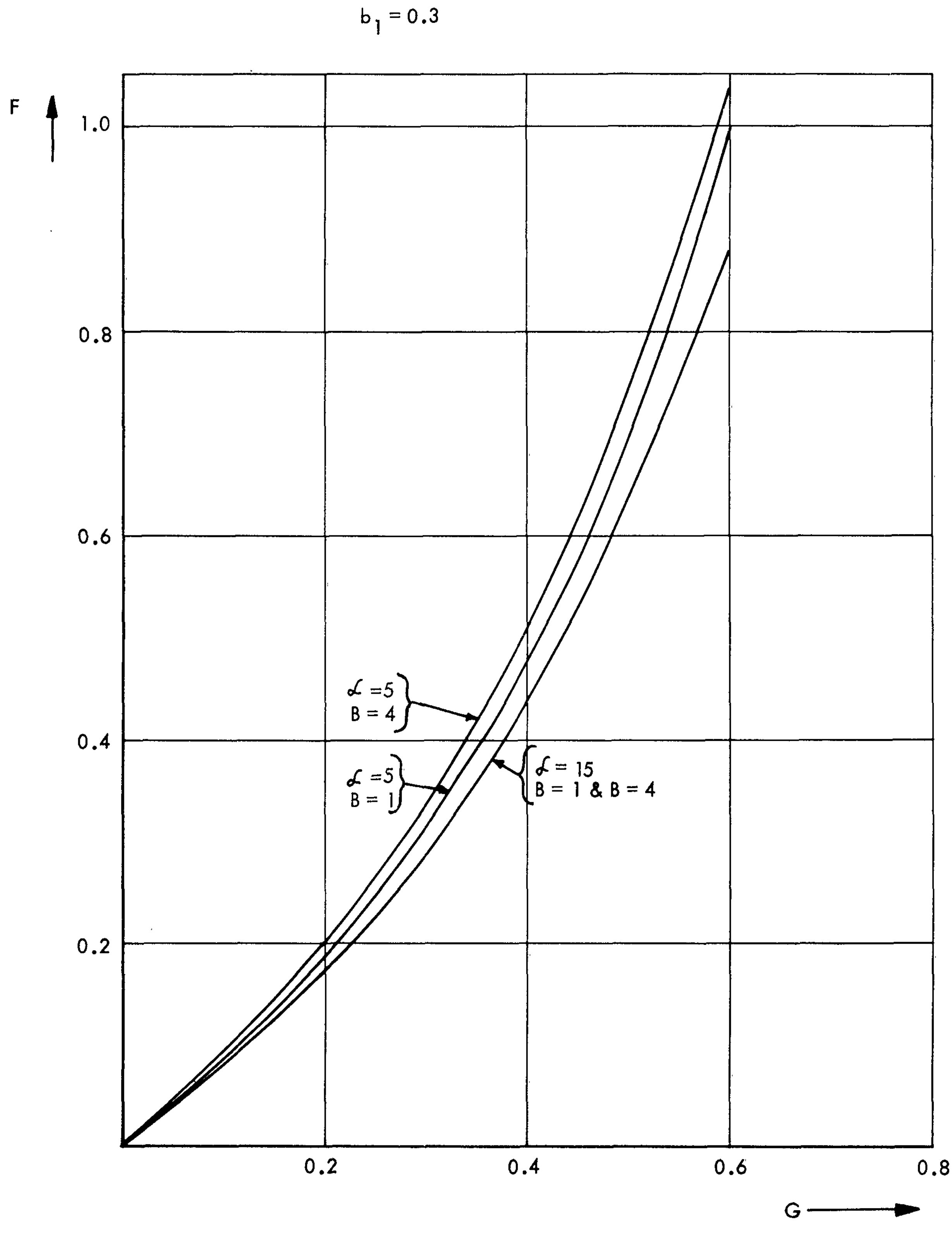


Figure 4. TENSION VS. FLOW RATE FOR SOURCE AT $b_1 = 0.3$

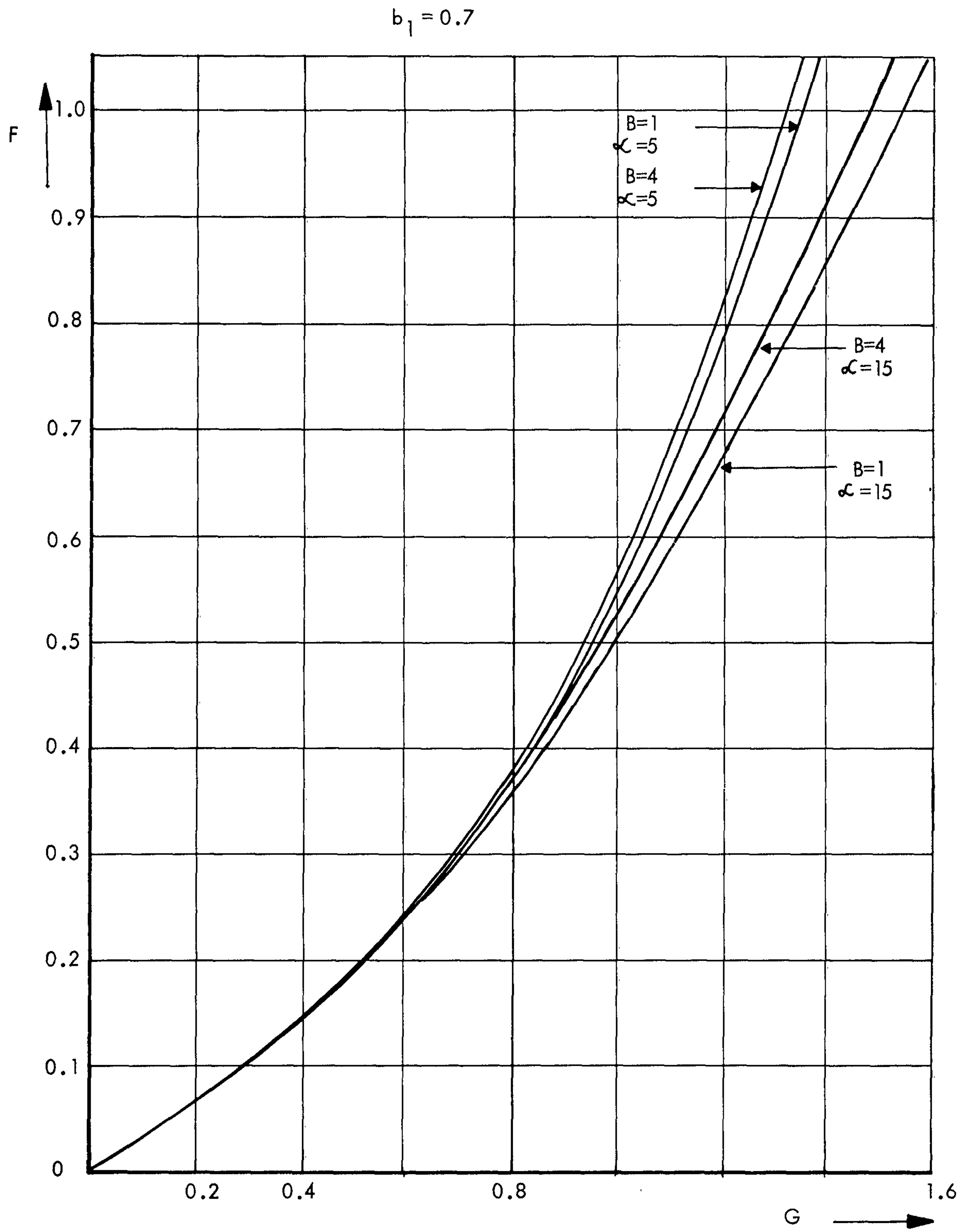


Figure 5. TENSION VS. FLOW RATE FOR SOURCE AT $b_1 = 0.7$

$b_1 = 0.85$

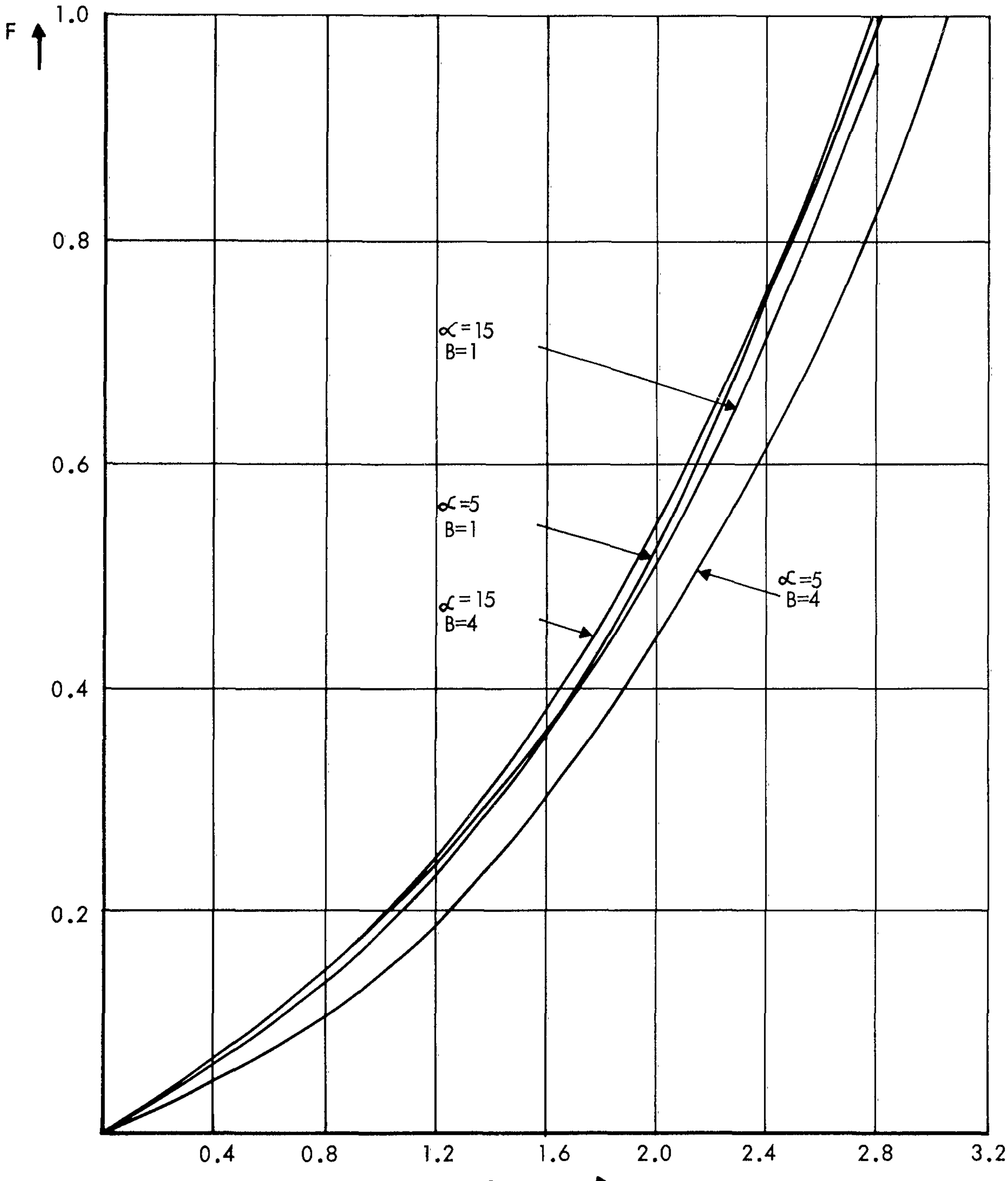


Figure 6. TENSION VS. FLOW RATE FOR SOURCE AT $b_1 = 0.85$

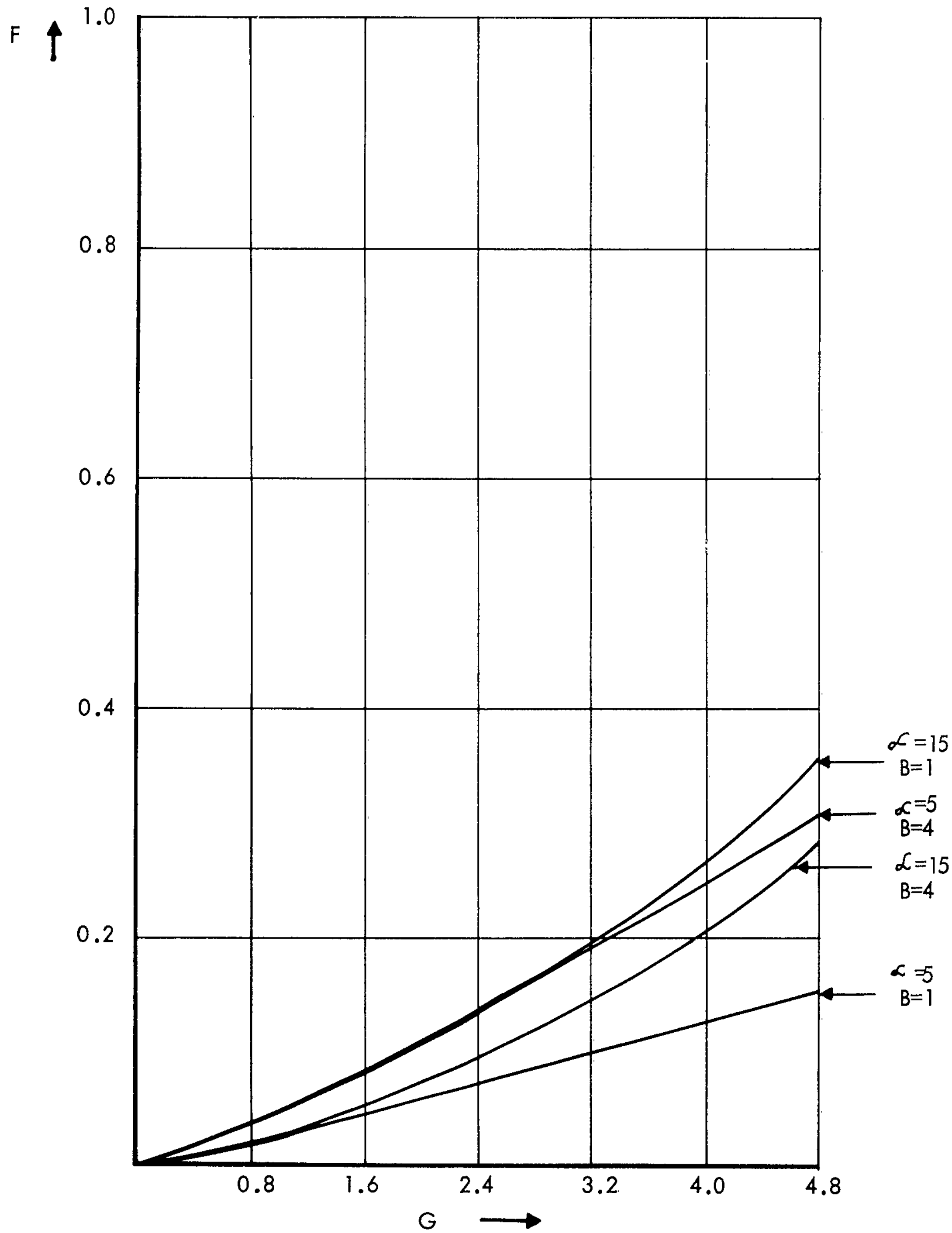
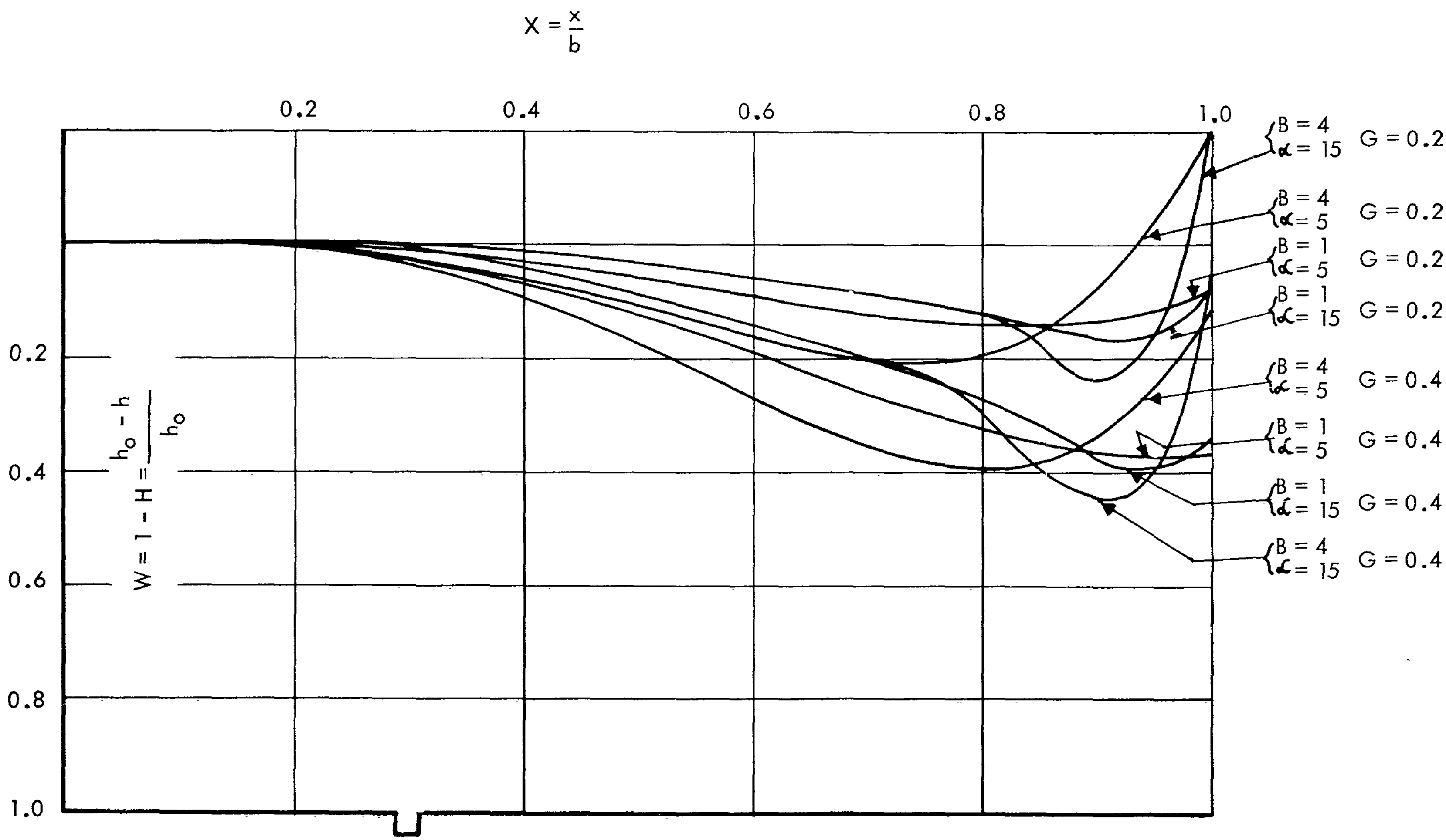


Figure 7. TENSION VS. FLOW RATE FOR SOURCE AT $b_1 = 0.95$



$b_1 = 0.3$

b and h_o are the tape half-width and the gap under the center line, respectively.

Figure 8. GAP PROFILE FOR SOURCE AT $b_1 = 0.3$

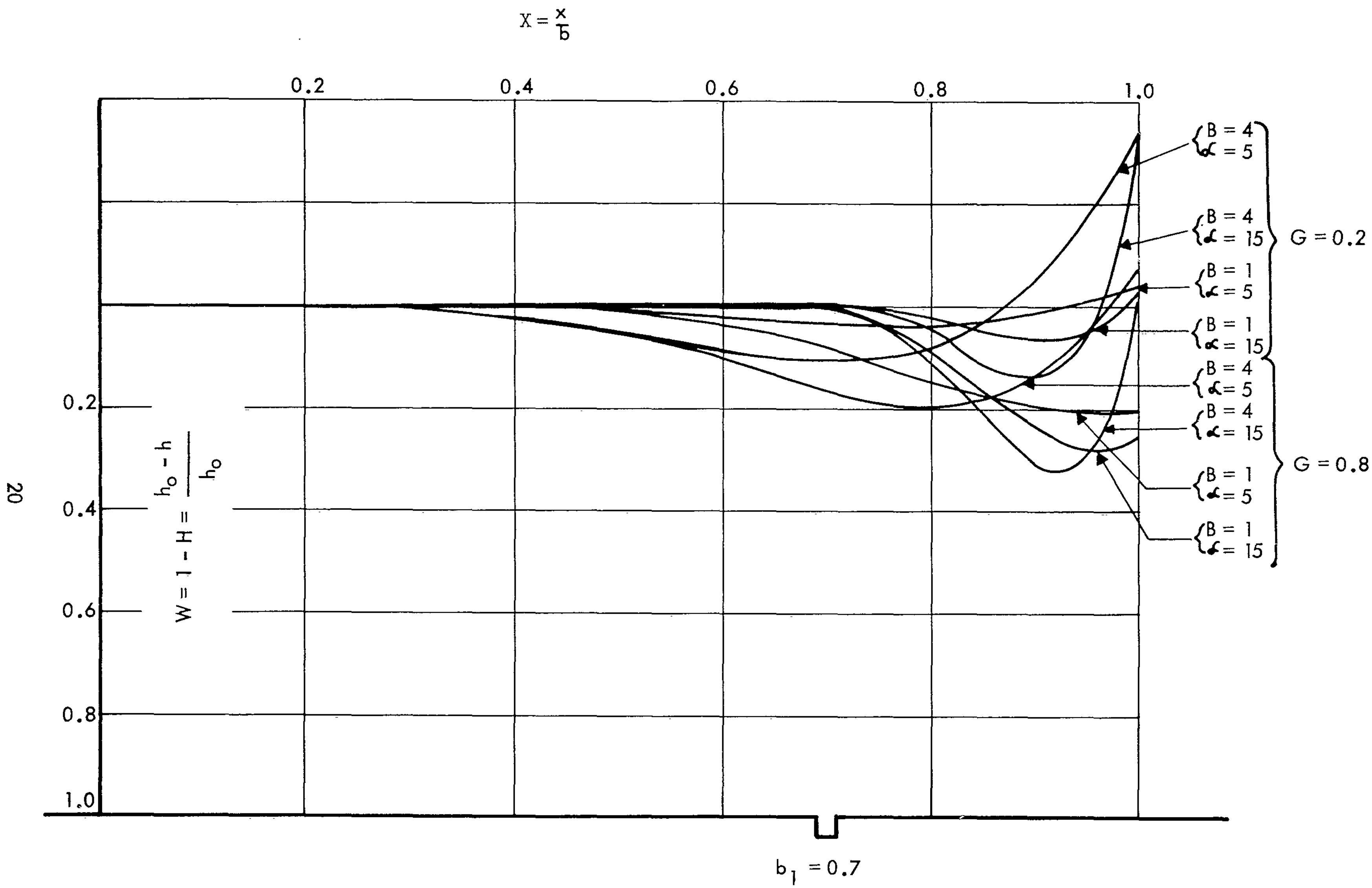


Figure 9. GAP PROFILES FOR SOURCE AT $b_1 = 0.7$

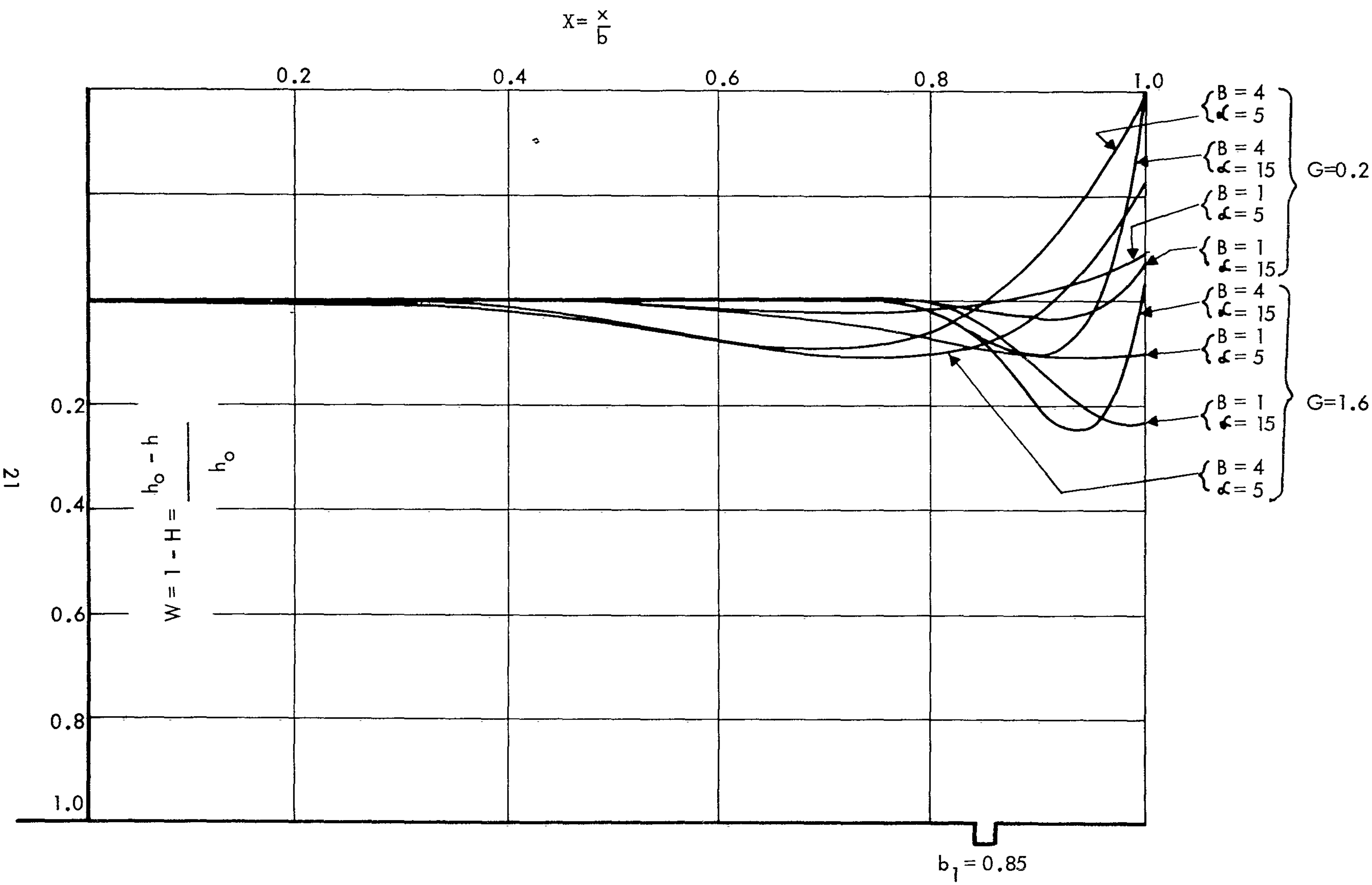


Figure 10. GAP PROFILES FOR SOURCE AT $b_1 = 0.85$

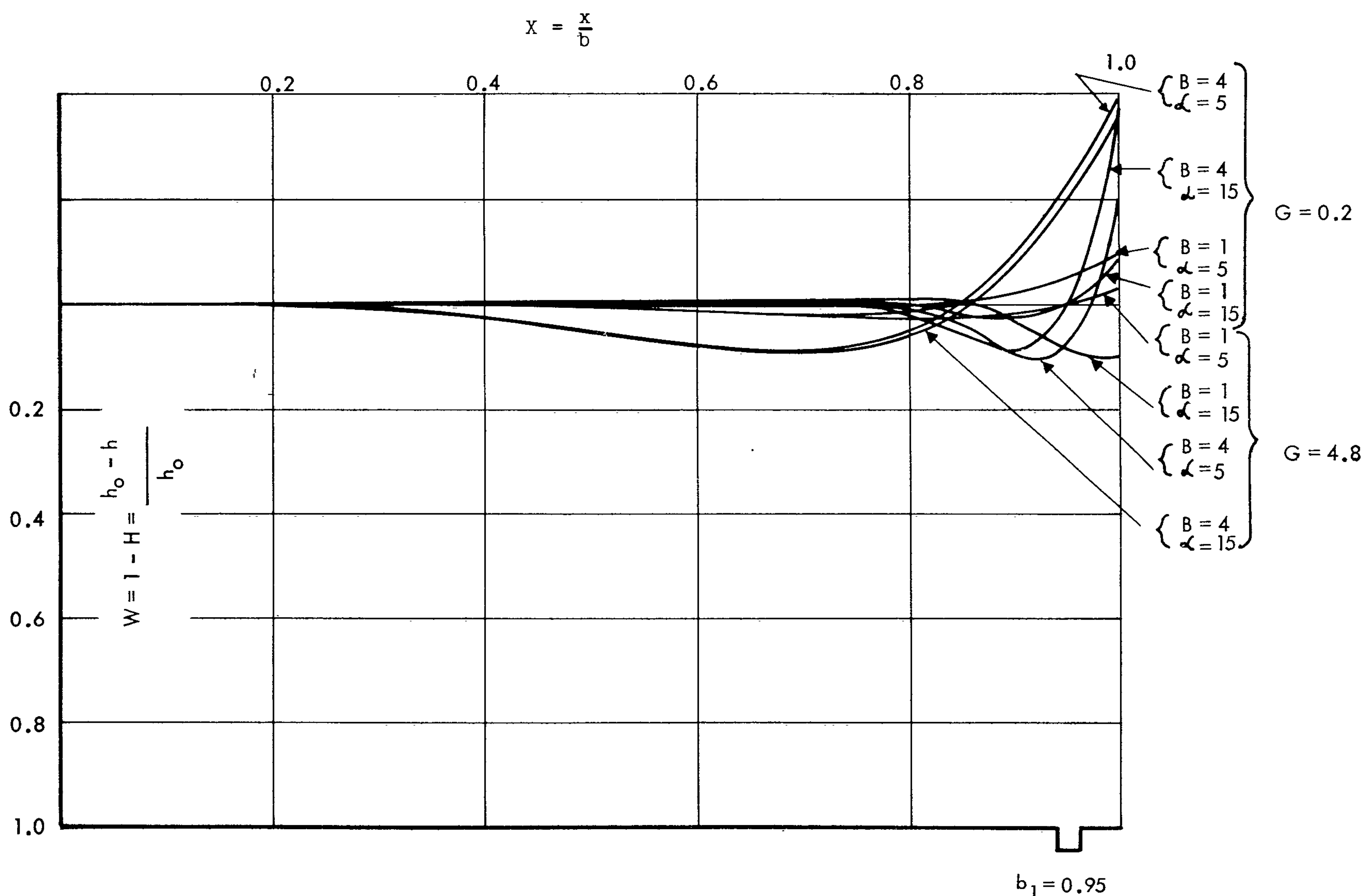


Figure 11. GAP PROFILES FOR SOURCE AT $b_1 = 0.95$

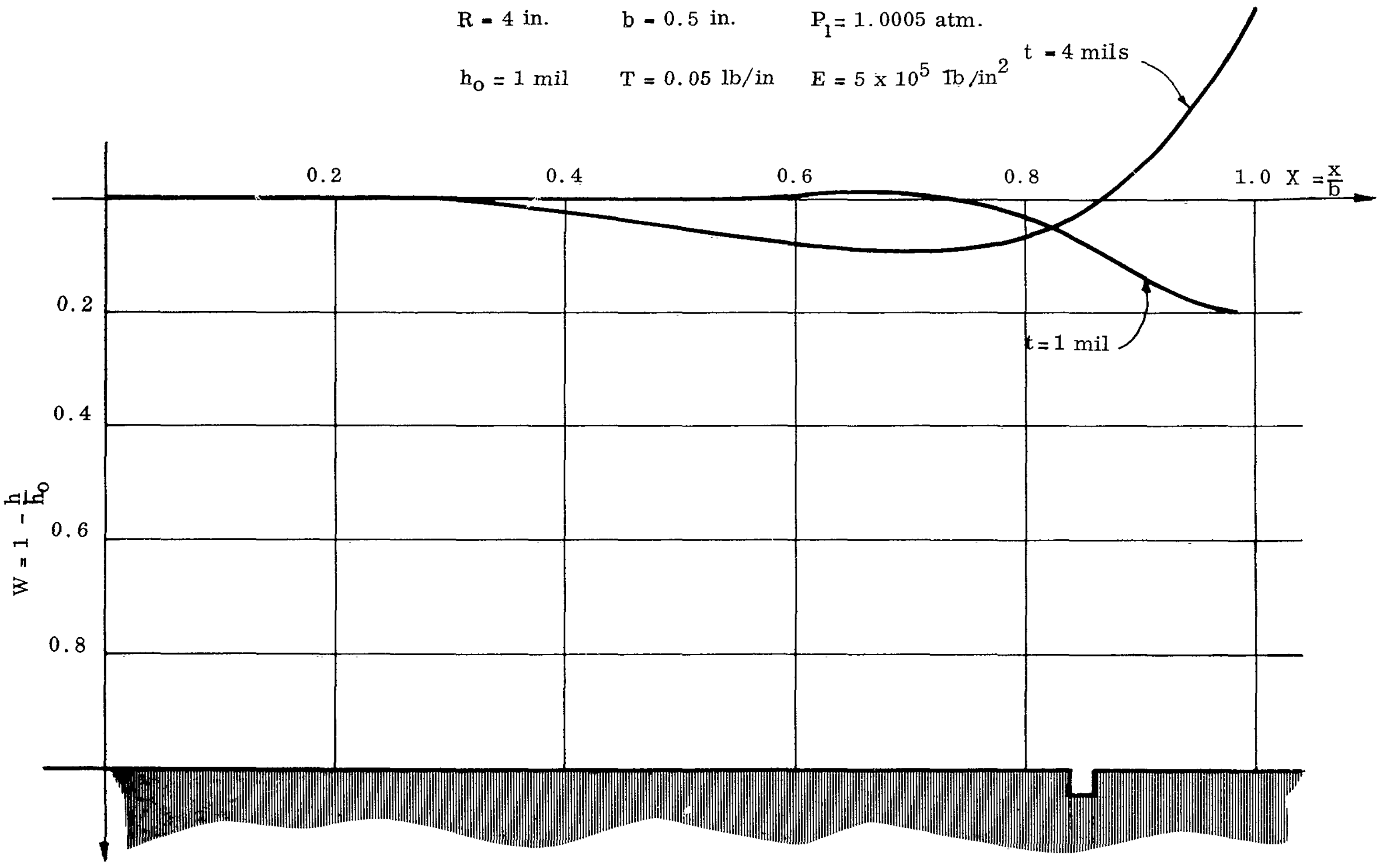


Figure 12. TYPICAL GAP PROFILES, b_1 = 0.85

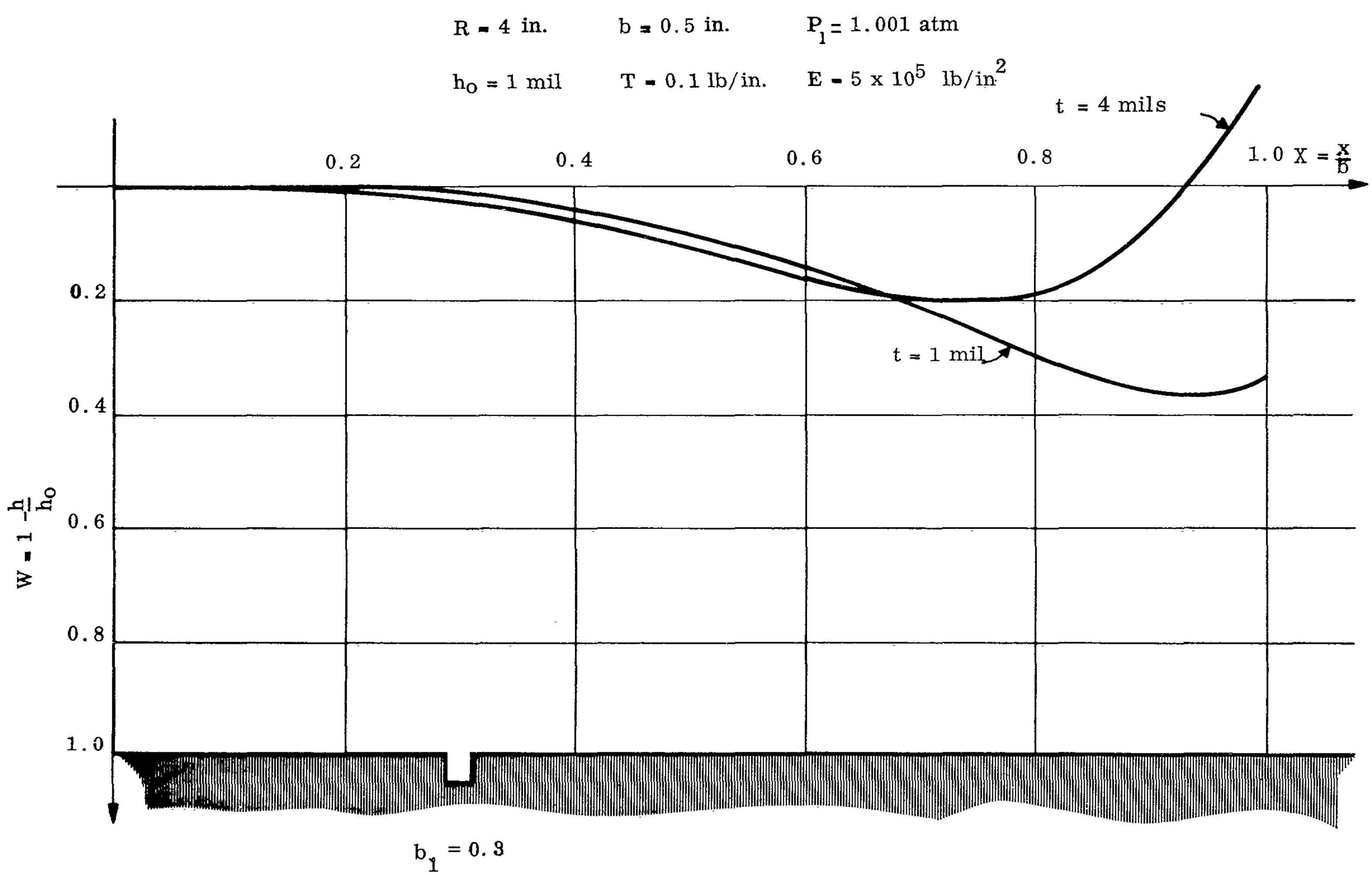


Figure 13. TYPICAL GAP PROFILES, $b_1 = 0.3$

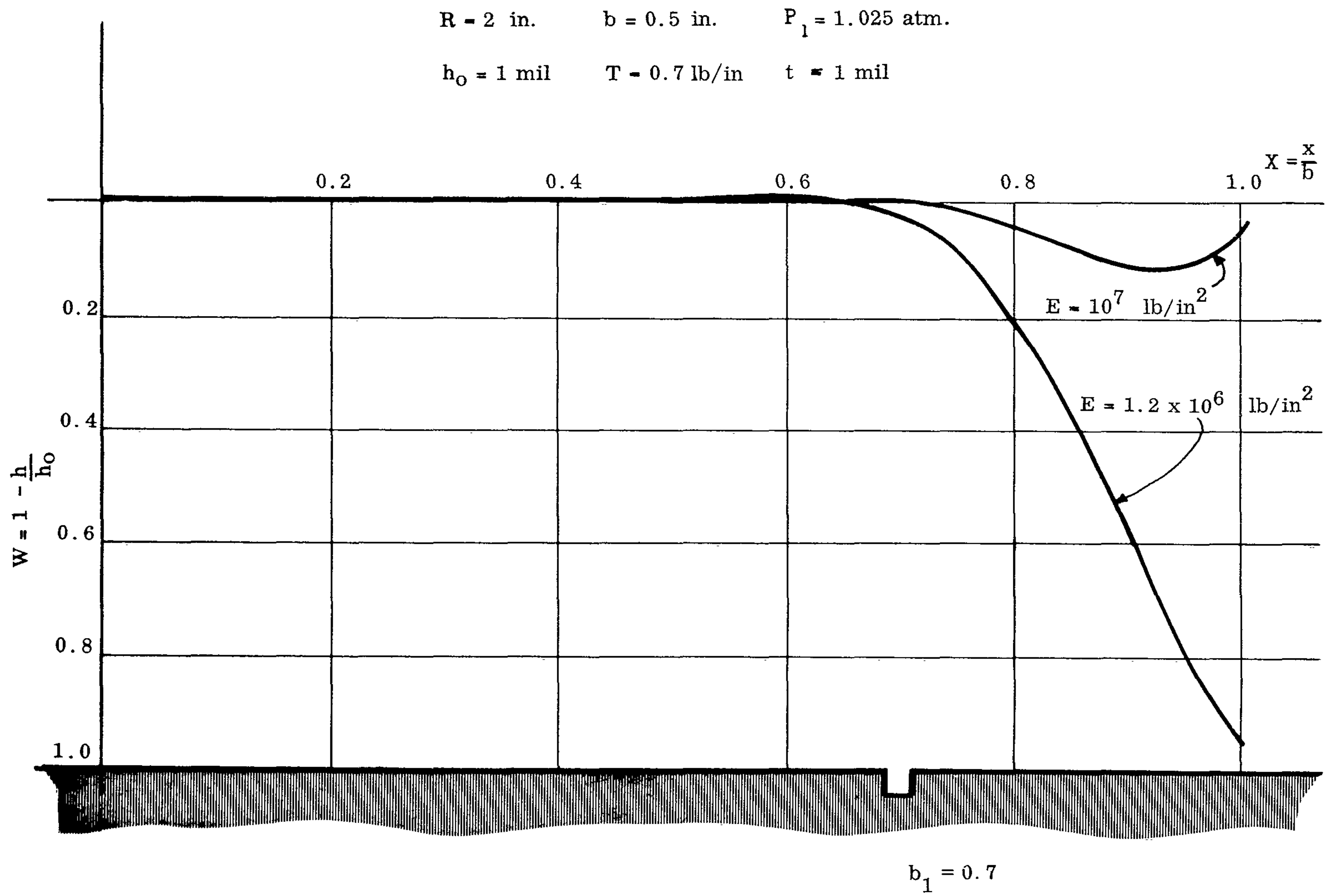


Figure 14. TYPICAL GAP PROFILES , b_1 = 0.7

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APPENDIX I

Equation (29) can be integrated to give

$$W = F - C(P_1-1) + C_1 \cosh \alpha X \cos \alpha X + C_2 \sinh \alpha X \sin \alpha X$$

$$+ C_3 \sinh \alpha X \cos \alpha X + C_4 \cosh \alpha X \sin \alpha X \qquad (1a)$$

Substituting into Equation (10), we find that

$$C_3 = C_4 = 0 \qquad (2a)$$

Equation (1a) holds for $X \leq b_1$. Outside this region, Equation (30) will be used as an approximate solution of Equation (17). Substituting Equation (17) into (11), letting $\alpha b_1 = \beta$, gives

$$K_1 n_1{}^2 e^{n_1 \alpha} + e^{n_2 \alpha} \{K_2 [(n_2{}^2 - n_3{}^2) \sin n_3 \alpha + 2 n_2 n_3 \cos n_3 \alpha]$$

$$+ K_3 [(n_2{}^2 - n_3{}^2) \cos n_3 \alpha - n_2 n_3 \sin n_3 \alpha] \}$$

$$+ e^{n_4 \alpha} \{K_4 [(n_4{}^2 - n_5{}^2) \sin n_5 \alpha + 2 n_4 n_5 \cos n_5 \alpha]$$

$$+ K_5 [(n_4{}^2 - n_5{}^2) \cos n_5 \alpha - 2 n_4 n_5 \sin n_5 \alpha] \}$$

$$= - \frac{\nu \sqrt{A}}{\sqrt{12(1-\nu^2)}} B, \qquad (3a)$$

$$K_1 n_1{}^3 e^{n_1\alpha} + e^{n_2\alpha} \{K_2 \ [n_2(n_2{}^2 - 3n_3{}^2) \ \sin n_3\alpha$$

$$+ n_3(3n_2{}^2 - n_3{}^2) \cos n_3\alpha] + K_3 \ [n_2(n_2{}^2 - 3n_3{}^2) \cos n_3\alpha$$

$$- n_3(3n_2{}^2 - n_3{}^2) \sin n_3\alpha]\} \quad + e^{n_4\alpha} \{K_4 [n_4(n_4{}^2 - 3n_5{}^2) \sin n_5\alpha$$

$$+ n_5(3n_4{}^2 - n_5{}^2) \cos n_5\alpha + K_5[n_4(n_4{}^2 - 3n_5{}^2) \cos n_5\alpha$$

$$- n_5(3n_4{}^2 - n_5{}^2) \sin n_5\alpha]\} \quad = 0 \qquad (4a)$$

Substituting Equations (1a) and (28) into Equations (22) and (30) gives

$$C_1 + F - C(P_1 - 1) = 0 \qquad (5a)$$

$$-\frac{1}{3} + K_1 e^{n_1\beta} + e^{n_2\beta}(K_2 \ \sin n_3\beta + K_3 \ \cos n_3\beta)$$

$$+ e^{n_4\beta}(K_4 \ \sin n_5\beta + K_5 \ \cos n_5\beta)$$

$$= \ C_1(\cosh\beta \cos\beta - 1) + C_2 \ \sinh\beta \ \sin\beta, \qquad (6a)$$

$$K_1 n_1 e^{n_1\beta}+e^{n_2\beta}[K_2(n_2 \sin n_3\beta+n_3 \cos n_3\beta)$$

$$+K_3(n_2 \cos n_3\beta-n_3 \sin n_3\beta)]$$

$$+e^{n_4\beta}[K_4(n_4 \sin n_5\beta+n_5 \cos n_5\beta) +K_5(n_4 \cos n_5\beta-n_5 \sin n_5\beta)]$$

$$= C_1(\sinh \beta \cos \beta -\cosh \beta \sin \beta) +C_2(\cosh \beta+\sinh \beta \cos \beta), \tag{7a}$$

$$K_1 n_1^2 e^{n_1\beta}+e^{n_2\beta} \{K_2[(n_2^2-n_3^2) \sin n_3\beta+2n_2 n_3 \cos n_3\beta]$$

$$+K_3[(n_2^2-n_3^2) \cos n_3\beta-2n_2 n_3 \sin n_3\beta]\}$$

$$+e^{n_4\beta}\{K_4[(n_4^2-n_5^2) \sin n_5\beta+2n_4 n_5 \cos n_5\beta]$$

$$+K_5[(n_4^2-n_5^2) \cos n_5\beta-2n_4 n_5 \sin n_5\beta]\}$$

$$= 2(C_2 \cosh \beta \cos \beta-C_1 \sinh \beta \sin \beta), \tag{8a}$$

$$K_1 n_1^3 e^{n_1\beta} + e^{n_2\beta}\ \{K_2[n_2(n_2^2-3n_3^2)\ \sin\ n_3\beta + n_3(3n_2^2-n_3^2)$$

$$\cos\ n_3\beta] + K_3[n_2(n_2^2-3n_3^2)\ \cos\ n_3\beta - n_3(3n_2^2-n_3^2)\ \sin\ n_3\beta]\}$$

$$+e^{n_4\beta}\ \{K_4[n_4(n_4^2-3n_5^2)\ \sin\ n_5\beta + n_5(3n_4^2-n_5^2)\ \cos\ n_5\beta]$$

$$+K_5[n_4(n_4^2-3n_5^2)\ \cos\ n_5\beta - n_5(3n_4^2-n_5^2)\ \sin\ n_5\beta]\}$$

$$= 2\ [-C_1(\sinh\ \beta\ \cos\ \beta + \cosh\ \beta\ \sin\ \beta)$$

$$+C_2(\sinh\ \beta\ \cos\ \beta - \cosh\ \beta\ \sin\ \beta)] \qquad (9a)$$

$$K_1 n_1^4 e^{n_1\beta} + e^{n_2\beta}\ \{K_2[(n_2^4-6n_2^2n_3^2+n_3^4)\ \sin\ n_3\beta$$

$$+4n_2n_3(n_2^2-n_3^2)\ \cos\ n_3\beta] + K_3[(n_2^4-6n_2^2n_3^2+n_3^4)\ \cos\ n_3\beta$$

$$-4n_2n_3(n_2^2-n_3^2)\ \sin\ n_3\beta]\} + e^{n_4\beta}\ \{K_4[(n_4^4-6n_4^2n_5^2+n_5^4)$$

$$\sin\ n_5\beta + 4n_4n_5(n_4^2-n_5^2)\ \cos\ n_5\beta] + K_5[(n_4^4-6n_4^2n_5^2+n_5^4)$$

$$\cos\ n_5\beta - 4n_4n_5(n_4^2-n_5^2)\ \sin\ n_5\beta]\}$$

$$= -4(C_1\ \cosh\ \beta\ \cos\ \beta + C_2\ \sinh\ \beta\ \sin\ \beta) \qquad (10a)$$

Equations (3a), (4a), and (6a) through (10a) form a set of seven simultaneous linear algebraic equations for C_1, C_2, and K_1 through K_5. Equation (5a) gives the relation between F and P_1. The above equations were solved for various values of A, B, and G, using the 7090 computer at David Taylor Model Basin.

APPENDIX II

The indicial equation,

$$\lambda^5 + A - 3AG = 0 \qquad (1b)$$

cannot be solved by algebraic means. Rolle's theorem gives us a restriction on the location of the roots. Defining K by

$$K \equiv \lambda^5 + A\lambda - 3AG \qquad (2b)$$

the maxima and minima of K are given by

$$\frac{dK}{d\lambda} = 5\lambda^4 + A = 0 \qquad (3b)$$

Since Equation (3b) has no real roots, K is a monotonically increasing function and must vanish at one point. For $\lambda = 0$, K is negative. Therefore, the real root of Equation (1b) is positive. Calling this root $\lambda_1 = \eta_1\alpha$, and setting $\lambda = \eta\alpha$, we can write Equation (1b) as

$$(\eta-\eta_1)(\eta^4+C_1\eta^3+C_2\eta^2+C_3\eta+C_4) = \eta^5+4\eta-12\,\frac{G}{\alpha} = 0 \qquad (4b)$$

where

$$C_1 = \eta_1, \quad C_2 = \eta_1{}^2, \quad C_3 = \eta_1{}^3, \text{ and } C_4 = \eta_1{}^4+4 \qquad (5b)$$

If we set $u = \frac{\eta}{\eta_1}$, the equation for the complex roots becomes

$$u^4 + u^3 + u^2 + u + 1 + \frac{4}{\eta_1{}^4} = 0 \qquad (6b)$$

The value for η_1 can be found from Equation (2b) by graphical means and can be refined by using the formula

$$\eta_1 = \frac{4\overline{\eta_1}^{\,5}+12\frac{G}{\alpha}}{5\overline{\eta_1}^{\,4}+4}. \tag{7b}$$

where $\overline{\eta_1}$ is the graphical solution. Equation (6b) can be solved exactly[6].

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